Maths for Computing
Tutorial 9

1. There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?

2. Prove that the number of partitions of n into at most k parts is equal to the number of partitions of $n + k$ into exactly k parts.

3. The number of all set partitions of $[n]$ into nonempty parts is denoted by $B(n)$, and is called the nth Bell number. We set $B(0) = 1$. Clearly, $B(n) = \sum_{i=0}^{n} S(n, i)$. Prove that $B(n+1) = \sum_{i=0}^{n} \binom{n}{i} B(i)$.

4. Prove that for all integers $n > 2$, the number $p(n) - p(n-1)$ is equal to the number of partitions of n in which the two largest parts are equal.

5. Prove that number of lattice paths (paths using right and up move only) in a grid of $n \times n$ from (0,0) (bottom-left) to (n, n) (top-right) is the nth Catalan number.

6. Prove that $C_n = \sum_{i=1}^{n} C_{i-1} C_{n-i}$, where C_n is the nth Catalan number for $n \geq 1$.